Procedure when we can divide d_x and d_y by a common factor --

- Divide the numbers in the column d_x by a common factor and name it d'_x.
- Divide the numbers in the column d_y by a common factor and name it d'_y
- Find squares of all the numbers in d'_x and name it d'^2_x
- Find squares of all the numbers in d'_y and name it d'^2_y
- Multiply d'_x and d'_y
- Find $\Sigma d'_x$, $\Sigma d'_y$, $\Sigma d'^2_x$, $\Sigma d'^2_y$, $\Sigma d'_x d'_y$

Use the formula

$$r = \frac{n\ \Sigma d'_x d'_y - \Sigma d'_x \Sigma d'_y}{\sqrt{n\Sigma d'^2_x - \left(\Sigma d'_x\right)^2}\ \sqrt{n\Sigma d'^2_y - \left(\Sigma d'_y\right)^2}}$$

Problem--

X -- 100, 200 , 300 , 400 , 500

Y -- 500 , 400 , 100 , 300 , 200

We have to find out r.

X	Y	(X-200)= d_x	(Y-400)= d_y	$\frac{d_x}{100} = d'_x$	$\frac{d_y}{100} = d'_y$	d'^2_x	d'^2_y	$d_x\,d'_y$
100	500	-100	100	-1	1	1	1	-1
200	400	0	0	0	0	0	0	0
300	100	100	-300	1	-3	1	9	-3
400	300	200	-100	2	-1	4	1	-2
500	200	300	-200	3	-2	9	4	-6
				$\Sigma d'_x=5$	$\Sigma d'_y= -5$	$\Sigma d'^2_x = 15$	$\Sigma d'^2_y= 15$	$\Sigma d_x d'_y = -12$

We know

$$r = \frac{n\Sigma\, d'_x\, d'_y - \Sigma\, d'_x\, \Sigma d'_y}{\sqrt{n\Sigma d'^2_x - \left(\Sigma\, d'_x\right)^2}\ \sqrt{n\Sigma d'^2_y - \left(\Sigma\, d'_y\right)^2}}$$

Here $\Sigma d'_x = 5$, $\Sigma d'_y = -5$, $\Sigma d'^2_x = 15$, $\Sigma d'^2_y = 15$, $\Sigma d'_x d'_y = -12$, n= 5

Put the values

$$r = \frac{5x\,(-12) \;-\; 5\,x\,(-5)}{\sqrt{5\,x\,15 - (5)^2}\ \sqrt{5x15 - (-5)^2}}$$

$$= \frac{-60+25}{\sqrt{75-25}\ \sqrt{75-25}}$$

$$= \frac{-35}{\sqrt{50}\ x\ \sqrt{50}}$$

$$= \frac{-35}{50}$$

= - 0.70

So ,the value of Karl Pearson's correlation coefficient is - 0.70

Finding out correct correlation coefficient

- For the correct value of r we need all the required values correct.
- Even a single figure will make the value of r wrong.
- If we compute the value of r and we know latter that some of the pairs are wrong then to find out the correct value of r we need to make some adjustments.
- In other words we will have to deduct wrong values and add the correct values.

Here is a problem --

13) In order to find the correlation coefficient between two variables X and Y from 12 pairs of observations the following calculations were made --

$\Sigma X= 30, \Sigma Y = 5 , \Sigma X^2 = 670 \ \Sigma Y^2 =285 , \Sigma XY = 334$.

On subsequent verification it was found that the pair (X=11 , Y=4) were copied wrongly , the correct values being (X=10 , Y = 14). Find the correct value of correlation co -efficient.

We should be clear that now all the values $\Sigma X= 30, \Sigma Y = 5 , \Sigma Y^2 =285 , \Sigma XY = 334$ given above are wrong .

Here we have the wrong values

X	Y	X^2	Y^2	XY
11	4	121	16	44

We have the correct values

X	Y	X^2	Y^2	XY
10	14	100	196	140

Now we have to find out

Correct ΣX, Correct ΣX^2, Correct ΣY, Correct ΣY^2, Correct ΣXY. To find them we will have to use the wrong values.

Here given

$\Sigma X = 30$, $\Sigma Y = 5$, $\Sigma X^2 = 670$ $\Sigma Y^2 = 285$, $\Sigma XY = 334$ are the wrong values.

Correct ΣX = Wrong figure of ΣX - wrong figure of X + Correct figure of X

= 30 - 11 + 10 = 29

Correct ΣY = Wrong figure of ΣY - wrong figure of Y + Correct figure of Y

= 5 -4 +14

= 15

Correct ΣX^2 = Wrong figure of ΣX^2 - (Wrong figure of X)^2 + (Correct figure of X)^2

$= 670 - (11)^2 + (10)^2$

$= 670 - 121 + 100$

$= 649$

Correct ΣY^2 = Wrong figure of ΣY^2 - (Wrong figure of Y)^2 + (Correct Figure of Y)^2

$= 285 - (4)^2 + (14)^2$

$= 285 - 16 + 196$

$= 465$

Correct ΣXY = Wrong figure of ΣXY - (Wrong figure of X . Wrong figure of Y) + (Correct figure of X . Correct figure of Y)

$= 334 - (11 \times 4) + (10 \times 14)$

$= 334 - 44 + 140$

$= 430$

So, the correct value of

$$r = \frac{\text{n correct } \Sigma XY \; - \text{correct } \Sigma X \text{ correct } \Sigma Y}{\sqrt{\text{n correct } \Sigma X^2 - \; (\text{correct } \Sigma X)^2} \;\; \sqrt{\text{ncorrect } \Sigma Y^2 \; - \; (\text{correct } \Sigma Y)^2}}$$

Here Correct $\Sigma X = 29$, Correct $\Sigma Y = 15$, Correct $\Sigma X^2 = 649$, Correct $\Sigma Y^2 = 465$, Correct $\Sigma XY = 430$

Put the values

$$r = \frac{12x430 \; - 29 \text{ X } 15}{\sqrt{12 \text{ x } 649 - \; (29)^2} \;\; \sqrt{12 \text{ X} 465 - \; (15)^2}}$$

$$= \frac{5160 \; - 435}{\sqrt{7788 - \; 841} \;\; \sqrt{5580 - 225}}$$

$$= \frac{4725}{\sqrt{6947} \quad \sqrt{5355}}$$

$$= \frac{4725}{83.35 \text{ X } 73.17}$$

$$= \frac{4725}{6098.72}$$

= + 0.77

Hence the correct correlation coefficient is + 0.77